The marginal distributions of lifetime variables which right censor each other

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Abstract

The failure time of a machine with two modes of failure can be modeled as the minimum of two failure times (each associated with one failure mode). However, this model is not unique, i.e., as shown by Tsiatis, the joint distribution of the failure time and failure mode of the machine does not characterise the joint distribution - nor the marginal distributions - of the failure times under the two competing risks. Peterson introduced pointwise sharp bounds for these marginals. Crowder recognized that these bounds are not functionally sharp and restricted the class of functions containing all feasible marginals. In another publication, the authors improved on these bounds via a functional characterisation of the set of feasible pairs of marginals. As it turns out, not only is each marginal distribution function bounded below by its corresponding lower bound, but also its density is bounded below by the derivative of this bound. We present a summary of these results, describe a statistical application to the construction of confidence bands, and concentrate on the analysis of its rate of consistency under specific examples.

1   Introduction

Let T be the failure time of a machine with two modes of failure and let I indicate its mode of failure, i. e., $I = \{1\}, \{2\}$ or $\{1, 2\}$ depending on whether failure of the machine was due to mode $1, 2$ or both simultaneously.