Chapter 10

Large deviations for diffusion processes in conuclear spaces and for SPDEs

In Chapter 8, we studied diffusion processes in conuclear spaces governed by stochastic differential equations. In these models, the drift term describes the deterministic evolution of the voltage potentials of the neuron while the diffusion term is added when random stimuli by electric impulses are present.

In this chapter, we derive a large deviation principle (LDP) for such processes when the diffusion term depends on a small parameter which tends to zero. The lower bounds are established by making use of the Girsanov formula in abstract Wiener space. The upper bounds are obtained by Gaussian approximation of the diffusion processes and by taking advantage of the nuclear structure of the state space to pass from compact sets to closed sets.

This chapter is organized as follows: We study the LDP for a class of random variables taking values in Banach spaces in Section 1. Then in Section 2, we apply our basic results to stochastic differential equations in the conuclear spaces investigated in Chapter 8. The material of this section comes from Xiong [60]. In Section 3, we present our results obtained in [32] for LDP of random field solution of SPDEs studied in Section 4.3. Finally, in Section 4, we specialize the results to stochastic reaction-diffusion equations.

10.1 LDP for a class of random variables

Stochastic differential equations or stochastic integrals can usually be regarded as random transformations of some Wiener processes. In this sec-