

Chapter 9

Asymptotic behavior of interacting system of stochastic differential equations on duals of nuclear spaces

The study of the limit of interacting n -particle diffusions was undertaken by McKean [39] whose work was followed by the papers of Hitsuda and Mitoma [15] and Shiga and Tanaka [49] among others.

This chapter is concerned with “propagation of chaos” problems for stochastic systems with an infinite number of degrees of freedom such as random strings or the fluctuation of voltage potentials of interacting spatially extended neurons. The latter is a more realistic model for large numbers of neurons in close proximity to one another and has provided the motivation for the work presented here which is novel in one respect: we consider Φ' -valued SDE's driven by Poisson random measures. The results for Φ' -valued interacting diffusions can be similarly derived. (See also Chiang, Kallianpur and Sundar [3])

The results obtained in this chapter are of interest beyond the neurophysiological applications that motivated them. It should be mentioned that the interaction considered here is mean field interaction which seems more applicable to phenomena in statistical physics. A type of interaction known as “parallel fiber interaction” has been used in connection with potentials of interacting neurons the consideration of which however, remains outside the scope of this book.