Chapter 8

Diffusion approximation and Φ'-valued diffusion processes

The study of SDE's in Chapter 6 is motivated by various practical problems. One of the applications is to the voltage potential of spatially extended neurons. The stimuli received by a neuron are the form of electrical impulses and are modelled by Poisson random measures. When the pulses arrive frequently enough and the magnitudes are small enough, it is reasonable to expect that the compensated Poisson random measures are approximated by Gaussian white noises in space-time and hence, the discontinuous processes of voltage potentials of spatially extended neurons governed by Poisson random measures are approximated by diffusion processes.

In this chapter, we study the existence and uniqueness for the solution of a diffusion equation on the dual of a CHNS. We shall consider it as the limiting case of the SDE's driven by Poisson random measures investigated in Chapter 6.

Let $(U, \mathcal{E})$ be a measurable space and μ^n a sequence of σ-finite measures on U. Let N^n be a sequence of Poisson random measures on $\mathbf{R}_+ \times U$ with characteristic measures μ^n. Let $A^n : \mathbf{R}_+ \times \Phi' \to \Phi'$ and $G^n : \mathbf{R}_+ \times \Phi' \times U \to \Phi'$ be two sequences of measurable mappings on the corresponding spaces. We consider a sequence of SDE's

$$X_t^n = X_0^n + \int_0^t A^n(s, X_s^n) ds + \int_0^t \int_U G^n(s, X_{s-}^n, u) \tilde{N}^n(duds) \qquad (8.0.1)$$

where $\{X_0^n\}$ is a sequence of Φ'-valued random variables and $\tilde{N}^n$ is the compensated random measure of N^n.

We prove that, under suitable conditions, the sequence of unique solutions of the SDE (8.0.1) converges in distribution to the unique solution of