# Chapter 7

# Stochastic models of environmental pollution

## 7.1 Introduction

Stochastic partial differential equations (SPDE's) arise from attempts to introduce randomness in a meaningful way into the study of phenomena hitherto regarded as deterministic. As examples one may cite recent research in chemical reaction-diffusions, neurophysiology or turbulence. In almost all cases, one takes as the starting point, the partial differential equations (PDE's) provided by the deterministic theories. The following PDE (given here in a somewhat simplified form) has been used in a deterministic study of pollution or water quality in a basin or reservoir:

$$D\Delta\phi - \left(V_1\frac{\partial\phi}{\partial x_1} + V_2\frac{\partial\phi}{\partial x_2}\right) - K\phi + Q = 0 \qquad (7.1.1)$$

with non-conductive boundaries. Here $\Delta$ is the Laplacian operator in a bounded domain in $\mathbf{R}^2$, $\phi(x_1, x_2) \geq 0$ is the water quality or chemical concentration at the point $(x_1, x_2)$ in the basin, $D$ is the diffusion coefficient, $V_j$ is the convective velocity in the $x_j$ direction, $K$ is the heat transfer coefficient and $Q \geq 0$ is the "load" pollutant issued from waste outfall.

The above equation is taken from a paper of T. Futagami, N. Tamai and M. Yatsuzuka [54] in which numerical methods for its solution are studied in detail. A PDE that corresponds to a transient or dynamic version of this model will be given in the next section (Eq. (7.2.1)).

Another model is the following river pollution model proposed by Kwakernaak [36] and studied by Curtain [6]: Suppose that the number of deposits in a section of the river of infinitesimal length $dx$ ($x$ being the distance coordinate along the river) behaves according to a Poisson process with rate parameter $\lambda(x)dx$ where $\lambda(x)$ is a given function. The number of deposits in