Chapter 6

Stochastic differential equations on Φ' driven by Poisson random measures

Stochastic differential equations (SDE's) on infinite dimensional spaces arise from such diverse fields as nonlinear filtering theory, infinite particle systems, neurophysiology, etc. In this chapter, we study SDE's on duals of nuclear spaces driven by Poisson random measures. Namely, we consider the following SDE

$$X_t = X_0 + \int_0^t A(s, X_s)ds + \int_0^t \int_U G(s, X_{s-}, u)\tilde{N}(duds) \tag{6.0.1}$$

on the dual of a CHNS Φ, where $A : \mathbf{R}_+ \times \Phi' \to \Phi'$, $G : \mathbf{R}_+ \times \Phi' \times U \to \Phi'$, $(U, \mathcal{E}, \mu)$ is a σ-finite standard measure space, N(duds) is a Poisson random measure on $\mathbf{R}_+ \times U$ with characteristic measure $\mu(du)$ and $\tilde{N}(duds)$ is the compensated random measure of N(duds). Motivated by neurophysiological problems, such equations were first considered by Kallianpur and Wolpert [27] [28] for finite dimensional equations (corresponding to the case when the neuron can be regarded as a single point) and for infinite dimensional linear equations. The general case was studied by Hardy, Kallianpur, Ramasubramanian and Xiong [13], most of the results of this chapter being taken from that paper.

The following assumption will be made throughout the rest of this book: There exists a sequence $\{\phi_i\}$ of elements in Φ, such that $\{\phi_i\}$ is a CONS in Φ_0 and is a COS in each space $\Phi_n, n \in \mathbf{Z}$.

Let $\phi_i^n \equiv \phi_i \|\phi_i\|_n^{-1}$, $n \in \mathbf{Z}$, $i \in \mathbf{N}^+$. It is easy to see that $\{\phi_i^n\}$ is a CONS in Φ_n.