Chapter 5

Stochastic differential equations in Hilbert space

Throughout this chapter, H will be a separable Hilbert space with inner product $< \cdot, \cdot >$ and norm $\|\cdot\|$. $L(H,H)$ will denote the class of all continuous linear operators on H and $L_2(H,H)$ the class of all Hilbert-Schmidt operators. For an operator $A \in L_2(H,H)$, the Hilbert-Schmidt norm will be denoted by $\|\cdot\|_2$.

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a given filtration $(\mathcal{F}_t)$ assumed to satisfy the usual conditions. Let (W_t) be an $(\mathcal{F}_t)$-cylindrical Brownian motion (c.B.m) on H and let (B_t) be an $(\mathcal{F}_t)$-adapted H-valued Brownian with covariance Σ (cf. Section 3.2 for definition).

5.1 Diffusion equations in Hilbert spaces

Suppose that $A : H \to H$ and $G : H \to L(H,H)$ are two continuous mappings. We consider the following SDE on H:

$$X_t = X_0 + \int_0^t A(X_s)ds + \int_0^t G(X_s)dB_s. \tag{5.1.1}$$

It is possible to establish a unique solution for (5.1.1) by making use of weak convergence techniques and by following the method which will be developed in Chapter 6, i.e., first we obtain a solution for the corresponding martingale problem by approximation and then get a weak solution by the representation theorems given in Chapter 3; finally we establish a unique strong solution by the Yamada-Watanabe argument. However, in this section, we shall adopt the approach given by Leha and Ritter [37] to establish a unique strong directly.