

Chapter 4

Stochastic partial differential equations

4.1 Introduction

Beginning with this chapter and throughout the rest of this monograph we will be considering examples from neurophysiology. There will, of course, be examples from other fields of applications. Nevertheless, different stochastic models of neuronal behavior have provided much of the motivation for the theory developed in the later chapters as well as examples of stochastic partial differential equations (SPDE's) that have random field solutions, solutions in Hilbert spaces or solutions in (conuclear) spaces of distributions. These examples will be considered in their proper contexts. It should also be pointed out that SPDE's where the driving processes are Poisson random measures arise naturally in the study of fluctuations of membrane potentials of neurons and can be used (as will be shown later) to derive diffusion approximations in infinite dimensional spaces. A similar approach might yield interesting results in other fields of application such as stochastic models of turbulence.

A brief description of the neurophysiological background may prove useful in understanding how some of the SDE's of this chapter and Chapter 8 are formulated.

In their seminal investigation in the early 1950's, Hodgkin and Huxley [16]) studied the electrical behavior of neuronal membranes and the role of ionic currents. They introduced a mathematical model for the flow of current through the surface membrane of the giant axon from a *Loligo Squid*. The partial differential equations which bear their name are nonlinear and have been at the center of a deterministic theory.

Although early stochastic models treated the neuron as a "point", in the neurophysiological literature, it has been well recognized that a neuron cell