Chapter 3

Stochastic integrals and martingales in Hilbert and conuclear spaces

From now on we shall be concentrating on two kinds of infinite dimensional spaces: a separable Hilbert space H and a conuclear space Φ', the strong dual of a CHNS Φ. Our aim in the present chapter is twofold: (1) To define martingales taking values in H and Φ' respectively.

While the study of such martingales (particularly H-valued martingales) is of importance in the general theory (see e.g. the books of Métivier [38] and da Prato and Zabczyk [45]), we confine our attention to discussing only those properties which are relevant to the theory of H or Φ'-valued SDE's.

(2) To introduce the definitions and study the basic properties of stochastic integrals taking values in H and Φ'. In contrast to finite dimensional stochastic calculus, we have three interested Brownian motions to consider: cylindrical Brownian motion, H-valued Brownian motion and Φ'-valued Brownian motion. We shall also define stochastic integrals with respect to a Poisson random measure.

We assume throughout that $(\Omega, \mathcal{F}, P)$ is a complete probability space with a right continuous filtration $\{\mathcal{F}_t\}_{t\geq 0}$. This chapter is organized as follows: After discussing some general properties of H-valued and Φ'-valued martingales, we introduce H-cylindrical Brownian motion (H-c.B.m), H-valued Brownian motion and Φ'-valued Wiener process. Then the stochastic integrals with respect to these processes will be defined and a representation theorem will be derived for H-valued and Φ'-valued continuous square-integrable martingales. Finally we define the stochastic integral with respect to Poisson random measure and give conditions for a Φ'-valued martingale to be represented as a stochastic integral with respect to a Poisson random measure. The two representation theorems will play important roles