

Chapter 2

Probability measures on topological spaces

As the duals of nuclear spaces are not metric spaces, to study Φ' -valued random variables or Φ' -valued stochastic processes we need to consider probability measures on general topological spaces. In Section 1 of this chapter, we first briefly recall some basic concepts about topological spaces. Then we establish some basic properties of Borel probability measures on general topological spaces. In Section 2 we study the weak convergence of Borel probability measures. In Section 3, we restrict ourselves to topological vector spaces and consider the Bochner functionals corresponding to cylinder measures. Finally in the last two sections we study two special topological spaces: $C([0, T], \Phi')$ and $D([0, T], \Phi')$ and probability measures. These two spaces will be our primary concern in the study of Φ' -valued stochastic processes with continuous sample paths and right-continuous sample paths respectively.

This chapter consists of basic material about probability measures on general topological vector spaces which we shall need in later chapters. For more detailed treatments we refer the reader to the books of Bilingsley [2], Ethier and Kurtz [9], Gel'fand and Vilenkin [12], Parthasarathy [43] and Xia [59]. Most of the material in Sections 4 and 5 is taken from Mitoma [41].

2.1 Probability measures on topological spaces

In this section we briefly present some basic concepts about topological spaces and consider probability measures on topological spaces. We shall study some special properties of the probability measures when the topology of the space has extra structure.