Chapter 1

Topological vector spaces

In the first two sections of this chapter we collect some necessary facts from functional analysis about topological vector spaces and their operator theory to make this book as self-contained as possible. Although we have provided all the proofs, the only exception being the proof of the spectral decomposition theorem, these two sections are not intended as an introduction to functional analysis for the beginner. We refer the reader who is interested in a more detailed treatment to standard textbooks on this topic such as Conway [5], Reed and Simon [47], Yosida [61].

In Section 3 we treat a special class of topological vector spaces: countable Hilbertian nuclear space and their dual spaces. As we shall see in later chapters, these spaces are very convenient for some practical problems and will play a major role in the course of this book. Most of the material in this section is taken from Kallianpur [23].

1.1 Topological vector spaces.

In this section we introduce the definition of a topological vector space (TVS) and state some basic properties of special classes of topological vector spaces such as Frèchet, Banach and Hilbert spaces for later use.

Definition 1.1.1 *A non-empty set X is called a* **topological vector space** *if it is a vector space with a topology compatible with the space structure, i.e., the following two maps*

$$(x, y) \in X \times X \mapsto x + y \in X \tag{1.1.1}$$

$$(\alpha, x) \in \mathbf{R} \times X \mapsto \alpha x \in X \tag{1.1.2}$$

are continuous.