PLAY-THE-WINNER DESIGNS, GENERALIZED PÓLYA URNS, AND MARKOV BRANCHING PROCESSES

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Abstract

The randomized play-the-winner rule, an adaptive design for clinical trials aimed at placing more patients on the better of two treatments, may be modeled as a generalized Pólya urn. The urn model may, in turn, be embedded in a Markov branching process, and results from the theory of these processes may then be used to prove results for the urn model, and hence for the randomized play-the-winner scheme. Under a mild condition for the success probabilities p_A and p_B for the two treatment arms, results from the theory of Markov branching processes show that the (random) probability of assignment to a given treatment is asymptotically normal; we extend this result to show that, under this same condition, the probability of assignment to a given treatment and the number of patients assigned to that treatment have a limiting bivariate normal distribution. Some generalizations of this result are discussed.

1. Introduction. Consider a clinical trial in which patients are accrued sequentially and immediately are assigned to treatment A or