On a Binomial Admissibility Problem
In Honor of Jack Hall on his 70th Birthday

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In this paper, X has a binomial (n,p) distribution, where n is known and p is unknown, $0 \leq p \leq 1$. Furthermore, let f be a given real valued continuous function on $[0,1]$. We will be interested in the question exactly when the "natural" estimator $T(X) = f(X/n)$ of $f(p)$ is admissible, always under squared loss.

1. Introduction. In this paper, X has a binomial (n,p) distribution, where n is known and p is unknown, $0 \leq p \leq 1$. Furthermore, let f be a given real valued continuous function on $[0,1]$. We will be interested in the question exactly when the "natural" estimator $T(X) = f(X/n)$ of $f(p)$ is admissible, always under squared loss. Special attention will be paid to the function

$$f_0(p) = \max(p, 1-p), \tag{1.1}$$

with associated estimator

$$T_o(X) = f_o(X/n) = \max(X/n, 1 - X/n). \tag{1.2}$$

It was stated by Johnson [4, p. 1586], and is easily shown, that:

(1.3)

i. If $n = 2m$ is even and $n \geq 6$ then T_o is inadmissible for f_o.

ii. If $n = 2m$ is even and $n \leq 4$ then T_o is admissible for f_o.

iii. If $n = 2m + 1$ is odd and $n \leq 7$ then T_o is admissible for f_o.

A main contribution of this paper is the following result.

THEOREM 1. *If $n = 2m + 1$ is odd and $n \geq 9$ then T_o is inadmissible as an estimator of $f_o(p)$.*

There are many other results. Some related papers are included in the list of references .

2. Auxiliary results. The following result is due to Johnson [4]. Here and below n, j, k, r and s usually denote integers.

THEOREM 2. *With $T(X)$ as a proposed estimator, of the form $T(X) = f(X/n)$, the following properties (i), (ii) are equivalent:*

i. $T(X)$ is an admissible estimator of $f(p)$ relative to squared loss.

ii. $T(X)$ admits a representation of the form