A Class of Parameter Functions for Which the Unbiased Estimator Does Not Exist

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Unbiased estimation is a popular criterion in small sample point estimation. However, there is limited knowledge about conditions under which the unbiased estimate does not exist. In this paper, in analogy to the binomial estimation, we give a class of parameter functions for which no unbiased estimator exists. The minimax bias estimators for these functions are obtained. The relationship between our results and the sample size and the interim review is commented.

1. Introduction Point estimation is a very important area in statistical inference. In small-sample estimation, a lot of attention is paid to unbiased estimation and a complete theory, the Blackwell-Rao and Lehmann-Scheffé theorems, about the uniformly minimum variance unbiased estimator (UMVU) has been developed. This topic appears in intermediate to advanced statistical inference textbooks, such as [3] and [12], as well as many lecture notes, such as [6]. According to the classic works by Lehmann and Scheffé [13], Halmos [7] and Bahadur [1], the condition of unbiasedness is generally a strong one. However, there is very limited knowledge about under what conditions the unbiased estimate does not exist. One well-known example is in binomial samples. From a sample of n Bernoulli trials with success probability π, only polynomials in π of degree no more than n can be estimated unbiasedly. There are few other examples for the non-existence of the unbiased estimator in statistical inference textbooks. Some approaches, primarily focusing on binomial estimation problems, are suggested by Bhattacharyya [2], Siraždinov [14], and Hall [5], when the unbiased estimator does not exist. We believe more examples for non-existence of the unbiased estimator will enrich both the theory of point estimation and statistical education.

A special problem comes from the practice of clinical trials. To determine the sample size, several parameters need to be assumed. For simplicity, we consider the one-sided one-sample z-test based on a sample from $N(\mu, 1)$, and testing H_0: $\mu = 0$ vs H_1: $\mu > 0$. Usually a target value μ_1 of the alternative is assumed, and the sample size is determined by the significance level and the power at this target value. Based on such a design, the whole power function can be obtained, so investigators can know what the power of the study is if the true difference is some value of μ. It is noted that the power of the study depends on the unknown parameter (treatment effect), and hence it is also unknown. Sometimes, this target value is given according to a well-established clinical significance, e.g., a test is required to have 80% power to detect H_1: $\mu = 1$, where people think $\mu \geq 1$ is clinically significant. However, it is not rare that people choose an estimated μ from a previous small study as the target. There could be two interpretations for using such an estimated μ in power calculation. One is treating this estimated value as a reasonable non-random