Inference for the Proportional Mean Residual Life Model

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We review the proportional mean residual life model for the analysis of reliability and survival data. In the single sample case with known baseline distribution the Fisher information matrix for the proportionality parameter is derived. A class of weighted ratio estimators is defined and it is shown that the right choice of weight function yields an asymptotically efficient estimator. It is conjectured that the methodology will extend to the two-sample case, i.e. with unknown baseline distribution.

1. Introduction Oakes and Dasu [15] introduced the *proportional mean residual life (PMRL)* model for the analysis of reliability and survival data . As its name suggests, in the two sample case this model implies that the mean residual life (MRL) functions for the two samples $e_j(x) = E(X_j - x|X_j > x)$, $(j = 1, 2)$ are in a constant (i.e. x-free) ratio θ,

$$e_2(x) = e(x; \theta) = \theta e_1(x). \tag{1}$$

We assume that the corresponding survivor functions $S_j(x) = P(X_j > x)$,$(j = 1, 2)$ are absolutely continuous.

In many ways the MRL function provides a more natural basis for the modeling of such data than the hazard function - the basis for Cox's proportional hazards model ([4]). The former summarizes the entire residual life distribution, whereas the latter relates only to the risk of immediate failure. In industrial reliability studies the MRL function may therefore be more important than the hazard function in the planning of strategies for maintenance and replacement. Demographers have used the *life expectancy* or *expectation of life* function $e(x) + x$ for centuries in studies of human populations. Hall and Wellner [10, 11] gave a detailed discussion of the properties of the MRL function. They characterized the class of distributions with linear MRL, $e(x) = ax + b$, and showed that the only continuous distributions with this property are the Pareto, exponential and a certain class of rescaled beta distributions.

The MRL function does have one serious disadvantage for statistical work. It is highly dependent on the tail behavior of the survivor function, and is therefore hard to estimate with precision, especially when no parametric form can be assumed. The model (1), if it is appropriate, would be expected to lead to substantial gains in efficiency in the estimation of each $e_j(x)$ for large x.

In this paper we study parametric and nonparametric methods for the analysis of data from (1). We concentrate mainly on the one-sample problem, where the baseline survivor function $S_1(x)$ is assumed known, but we also consider the the two-sample problem where both functions are unknown. In Section 2 we review some known results concerning the MRL and its sample estimate, and present conditions for the existence of a PMRL family. Section 3 considers maximum likelihood estimation of θ in the one-sample case, derives a simple expression for Fisher's