# Gaussian White Noise Models: Some Results for Monotone Functions

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Gaussian white noise models have become increasingly popular as a canonical type of model in which to address certain statistical problems. We briefly review some statistical problems formulated in terms of Gaussian "white noise", and pursue a particular group of problems connected with the estimation of monotone functions. These new results are related to the recent development of likelihood ratio tests for monotone functions studied by [2]. We conclude with some open problems connected with multivariate interval censoring.

**1. Introduction.** This paper briefly reviews some of the recent research involving white noise models, and then develops some new results for statistical inference about monotone functions in the presence of white noise. The themes developed here differ substantially from the talk (on *Semiparametric Models with Sum Tangent Spaces*) which I presented at the Rochester meeting held in the Fall of 1999 in honor of Jack Hall's 70th birthday. The subject of that talk was more directly connected with my joint work with Jack in the late 70's and early 80's on semiparametric models. But one thing I learned from Jack Hall during my time at Rochester was not to become too fixed on any one problem or point of view, and that often a research problem can only be thoroughly understood by coming at it from several different perspectives or standpoints.

Jack Hall had an enormous impact on my development as a young statistician. Jack's continued interest in research and enthusiasm for good problems has been an inspiration.

In Section 2, we briefly review a slice of the past and current research work on "white noise models". In Section 3, we present some results on estimation of a monotone function observed "in white noise", and study a canonical version of the problem which arises repeatedly in the asymptotic distribution theory for nonparametric estimators of monotone functions. Section 3 carries through an analogous estimation problem in which some additional knowledge of the monotone function is available, namely its value at one point. This arises naturally when addressing the problem of finding a likelihood ratio test of the hypothesis $H : f(t_0) = \theta_0$ where $f$ is monotone. The resulting likelihood ratio test statistic is introduced and studied in Section 5. Section 6 raises some further questions and problems. In particular we pose a problem concerning estimation of a monotone function of two variables subject to white noise on the plane (Brownian sheet) with a connection to multivariate interval censoring.

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