AFFINE INVARIANT LINEAR HYPOTHESES FOR THE MULTIVARIATE GENERAL LINEAR MODEL WITH VARMA ERROR TERMS

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Affine invariance is often considered a *natural* requirement when testing hypotheses in a multivariate context. This invariance issue is considered here in the problem of testing linear constraints on the parameters of a multivariate linear model with VARMA error terms. We give a characterization of the collection of null hypotheses that are invariant under the group of affine transformations, hence compatible with a requirement of affine invariant testing. We comment the results and discuss some examples.

1. Introduction

Affine invariance/equivariance often is considered a natural requirement in multivariate statistical inference. The rationale for such a requirement is that the data at hand, or the noise underlying the model, should be treated as intrinsically multivariate objects, irrespective of any particular choice of a coordinate system. This requirement plays a fundamental role in most recent developments in the area of robust multivariate analysis, where the concepts of spatial quantiles, spatial signs, spatial ranks, location or regression depth and contours, ..., all refer to either rotational or affine invariance/equivariance (see for instance Oja (1999) for a recent review). In such a context, reasonable testing procedures should be invariant—as soon, of course, as the null hypothesis itself is invariant.

Robust multivariate inference so far has been developed essentially for independent observations (location and regression models, MANOVA, principal components, ...). However, testing methods based on multivariate signs and ranks (more precisely, *interdirections* and the so-called *pseudo-Mahalanobis ranks*) recently have been extended (Hallin and Paindaveine, 2002a–c) to time-series problems. More specifically, these papers are treating the problem of testing linear hypotheses in the multivariate general linear model with VARMA error terms (equivalently, a VARMA model with linear trend) described below. As test statistics based on interdirections and pseudo-Mahalanobis ranks are automatically invariant under linear transformations, a preliminary question naturally arises: are affine invariance properties in this setting still meaningful? And, in case they are, which are the invariant null hypotheses?

This question, which is of a purely algebraic nature, is addressed here in full generality, and the class of invariant linear hypotheses, hence the class of testing problems that qualify for being treated by means of interdirections