SOME EFFICIENCY COMPARISONS FOR ESTIMATORS FROM QUASI-LIKELIHOOD AND GENERALIZED ESTIMATING EQUATIONS

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This paper is concerned with general methods for making efficiency comparisons for estimators with the aid of matrix inequalities. It is shown that a unified approach is possible for several distinct cases in the general regression model, one involving quasi-likelihood estimation and the other two generalized estimating equations. The paper includes some specific comparisons between an ordinary least squares estimator and the best linear unbiased estimator for an error component model. We also present some numerical examples.

1. Introduction

It is always interesting and useful to make efficiency comparisons in estimation in statistics. For example, simple estimators which are suboptimal may suffer little loss in efficiency relative to likelihood based estimators which are difficult to use. For regression models and inferences, Heyde (1997) and Rao and Rao (1998) present various results. In the present paper, we focus on the following cases for which we give a unified treatment:

1. Heyde (1989) introduces composite quasi-likelihood estimators (QLE). The comparison ensures that composition is generally advantageous. Heyde and Lin (1992) and Heyde (1997) study quasi-likelihood estimators for the general linear model. Two alternatives are $\hat{\theta}_A$ and $\hat{\theta}_{QS(V)}$, the latter being prefered on efficiency grounds.

2. Balemi and Lee (1999) make an application to clustered binary regression in the context of generalized estimating equation (GEE) (see Liang and Zeger, 1986; McCullagh and Nelder, 1989, Section 9.4), and include an efficiency comparison involving a working correlation matrix R and the correct correlation matrix R_0.

3. Wang and Shao (1992) and Liu and Neudecker (1997) study Σ, the asymptotic variance matrix of an estimator $\hat{\beta}_I$ under the independence working assumption studied by Liang and Zeger (1986). Wang and Shao (1992) give Σ an upper bound in the Löwner order sense. Liu and Neudecker (1997) give both the determinant $|\Sigma|$ and the trace $tr\Sigma$ an upper bound.

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