ON ESTIMATING THE PERIOD OF A CYCLIC POISSON PROCESS

ROELOF HELMERS
CWI

I WAYAN MANGKU
Bogor Agricultural University

We propose and investigate a simple nonparametric estimator of the period of a cyclic Poisson process. It is assumed that only a single realization of the Poisson process is observed in a bounded window. We prove consistency and establish a rate of convergence of the proposed estimator when the size of the window expands.

1. Introduction and main result

Let X denote a cyclic Poisson point process defined on a probability space $(\Omega, \mathcal{A}, P)$, with absolutely continuous σ-finite mean measure μ w.r.t. Lebesgue measure ν and with (unknown) locally integrable intensity function $\lambda\colon \mathbb{R} \to \mathbb{R}^+ \cup \{0\}$, i.e., for any bounded Borel set B, we have $\mu(B) = \int_B \lambda(s)\,ds < \infty$. In addition, λ is cyclic (with period τ), i.e., for some $\tau \in R^+$

$$\lambda(s + k\tau) = \lambda(s) \tag{1.1}$$

for all $s \in \mathbb{R}$ and $k \in \mathbb{Z}$. The period τ is assumed to be unknown.

Suppose that, for some $\omega \in \Omega$, a single realization $X(\omega)$ of the Poisson point process X is observed, though only in a bounded interval (called window) $W \subset \mathbb{R}$. Since λ is locally integrable, the Poisson point process X always places only a finite number of points in any bounded subset of $\mathbb{R}$. In order to investigate the consistency of an estimator of τ we let the window W depend on "time" $n = 1, 2, \ldots$, in such a way that $|W_n| \to \infty$, as $n \to \infty$, where $|W_n|$ denotes the size (or Lebesgue measure) of W_n. In this set-up, a necessary condition for the existence of a consistent estimator (of τ) is that $\int_{\mathbb{R}} \lambda(s)\,ds = \mathrm{E}\,X(\mathbb{R}) = \infty$, which implies that P almost surely the point pattern $X(\omega)$ contains infinitely many points (cf. Rathbun and Cressie, 1994). Note that for cyclic λ the requirement $\int_{\mathbb{R}} \lambda(s)\,ds = \infty$ is automatically satisfied, provided the global intensity $\theta = \tau^{-1} \int_0^\tau \lambda(s)\,ds$ of the process X is positive. Therefore we will assume throughout that $\theta > 0$.

The aim of this paper is to propose and investigate a simple nonparametric estimator $\hat{\tau}_n$ of the period τ of a cyclic Poisson process X, using a single realization $X(\omega)$ of X, observed in the window W_n. Let Θ denote the parameter space, $\tau \in \Theta$, and let Θ be a bounded open interval in $\mathbb{R}^+$, such that no multiple of τ is contained in Θ. Our estimator $\hat{\tau}_n$ of τ is obtained as follows: for any $\delta \in \Theta$, define

$$Q_n(\delta) = \frac{1}{|W_n|} \sum_{i=1}^{N_{n\delta}} \left(X(U_{\delta,i}) - \frac{1}{N_{n\delta}} \sum_{j=1}^{N_{n\delta}} X(U_{\delta,j}) \right)^2 \tag{1.2}$$