GENERAL SADDLEPOINT APPROXIMATION METHODS FOR SMOOTH FUNCTIONS OF M-ESTIMATES WITH BOOTSTRAP APPLICATIONS

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Procedures are developed and implemented for computing general saddlepoint approximations for statistics defined by estimating equations and functions of these. Our approach is based on the fact that such statistics can be approximated as finite linear combinations of products of centered, normalized averages, and that cumulants of such approximants may be evaluated to any desired accuracy. The resulting approximations are useful in a wide variety of applications and may be computed using computer algebra routines. The application of these procedures to replace bootstrap sampling (in the case when the underlying distribution is an empirical cdf) is discussed. Mathematica code implementing these procedures is available at: `http://www.utstat.utoronto.ca/david/expand.dist.nb`

1. Introduction

This paper is concerned with the development and implementation of general saddlepoint approximations to the distributions of statistics belonging to a broad class, namely those which may be represented as smooth functions of M-estimators of identically and independently distributed variables leading to empirical versions related to bootstrap distributions. This work follows the earlier developments of Young and Daniels (1991) and DiCiccio, Martin and Young (1992a,b). Symbolic and numerical implementation of the procedures is carried out using Mathematica (Wolfram, 1988) and is based on some extensions of the general saddlepoint approximation method of Easton and Ronchetti (1986). See also Barndorff-Nielsen and Cox (1989), Wang (1992), Jing, Feuerverger and Robinson (1994) and Hu and Kalbfleisch (2000). Our approach relies fundamentally on the fact that, in general, such statistics can be approximated arbitrarily well as finite linear combinations (of a certain form) of products of centered, normalized averages, and that in turn, the cumulants of such approximants are straightforward to evaluate to any desired accuracy.

In Sections 2–4, the main methodological and computational details are presented. Specifically, in Section 2 it is shown firstly that a single M-estimator may be approximated in the form (2.9)–(2.10) and consequently that any smooth function of M-estimators may be approximated as in (2.13)–(2.15). We then consider a typical approximation (2.14) of this type and in Section 3 discuss how its cumulant structure may be estimated to any

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