TWO TYPES OF INFECTIVES AMONG HOMOGENEOUS IVDU SUSCEPTIBLES

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This paper extends some classical random allocation models for intravenous drug users (IVDUs) to the case where the infectives may be of different types while the susceptibles are homogeneous. A general recursive equation for the probability generating function of the process is derived when there are only two infective types, and the first few pgfs obtained explicitly. Recursive equations for the expectations of new infectives of the two types are found, and a procedure for deriving these explicitly outlined. A simple example is provided, illustrating the difference between this case and that where both susceptibles and infectives are homogeneous.

1. Introduction

Some years ago, Gani (1991, 1993) applied a random allocation model to the problem of needle sharing among IVDUs; in this problem, both susceptibles and infectives were assumed to be homogeneous. The model was later used by Gani and Yakowitz (1993) to describe the spread of HIV among IVDUs. In a more recent paper, Gani (2002) has extended the model to the case where susceptibles are heterogeneous while infectives are homogeneous, and derived the expectations of the numbers of new infectives of different types generated after an exchange of needles. Some asymptotic results for these were also obtained.

The present paper is concerned with the case where the susceptibles are homogeneous, but the infectives may be heterogeneous, and in particular where they are of two types. Before we discuss this model, however, we remind the reader of some results for the simple case where there are n susceptibles and i infectives, both homogeneous. We shall assume that, after an exchange of needles, all susceptibles receiving needles from one or more infectives become newly infected. If there are s of these, then we may write their probability as

$$\mathbf{p}_s(i,n) = P\{s \text{ new infectives} \mid i \text{ infectives and } n \text{ susceptibles initially}\},$$

which satisfies the recursive equation

$$\mathbf{p}_s(i+1,n) = \mathbf{p}_{s-1}(i,n)\left(1 - \frac{s-1}{n}\right) + \mathbf{p}_s(i,n)\frac{s}{n} \tag{1.1}$$

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