EXPLICIT SOLUTIONS IN A SMOOTH CHANGE PROBLEM

ANDREW L. RUKHIN
University of Maryland, Baltimore County

A setting of the change-point estimation problem with a "smooth" change is investigated. A special class of such models, where the change is defined as a gradual shift in the parameter, is considered. We show that these models can be analyzed via explicitly given procedures. It is demonstrated that M-estimators can consistently estimate the change in the parameter structure as well as the additional unknown parameter. Under mild regularity assumptions, asymptotic normality and asymptotic efficiency of these procedures are established.

1. Introduction

The traditional setting of the change-point problem assumes that the change is abrupt and permanent. Clearly there are situations where this assumption is not met, and it is desirable to estimate the time moment at which the stationary character of observations has changed, or to test the hypothesis that such a change has indeed occurred. The possibility of simultaneous consistent estimation of the parameters characterizing the change of means has been demonstrated by Yao and Au (1989), who considered nonlinear regression models with step functions describing the evolution of the mean. Huang and Chang (1993) studied models with smooth change intervention when the observations during the change period have the distributions which are mixtures of the pre- and after- change distributions. Rukhin and Vajda (1997) investigated a general nonlinear regression model for the change in the mean function. See Brodsky and Darkhovsky (1993) and Csorgo and Horvath (1998) for further references.

Here we study a model with a smooth change in the parameter. As in Husková (1996) minimum contrast estimators are used for estimation of the change rate.

2. The model

The following model relates the change estimation to nonlinear parameter estimation. Consider a parametric family of probability distributions P_θ, $\theta \in \Theta$, $\Theta \subset R^p$ with Θ being an open convex subset of Euclidean space. Let $f(\cdot \mid \theta)$ denote the corresponding densities. We assume regularity conditions

Keywords and phrases: asymptotic efficiency; change-point problem; consistency; exponential family; information matrix; M-estimators.

AMS subject classifications: Primary 62F12; Secondary 62E20.