ON RANDOM WALKS AND DIFFUSIONS RELATED TO PARRONDO'S GAMES

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In a series of papers, G. Harmer and D. Abbott study the behavior of random walks associated with games introduced in 1997 by J.M.R. Parrondo. These games illustrate an apparent paradox that random and deterministic mixtures of losing games may produce winning games. In this paper, classical cyclic random walks on the additive group of integers modulo m, a given integer, are used in a straightforward way to derive the strong law limits of a general class of games that contains the Parrondo games. We then consider the question of when random mixtures of fair games related to these walks may result in winning games. Although the context for these problems is elementary, there remain open questions. An extension of the structure of these walks to a class of shift diffusions is also presented, leading to the fact that a random mixture of two fair shift diffusions may be transient to $+\infty$.

1. Introduction

The purpose of this paper is to study a family of random walks that include those arising in the games devised by J.M.R. Parrondo in 1997 to illustrate the apparent paradox that two "losing" games can result in a "winning" game when one alternates between them. We refer the reader to Harmer and Abbott (1999a,b), Harmer, Abbott and Taylor (2000) and Harmer, Abbott, Taylor and Parrondo (2000) in which Parrondo's paradox is discussed, large simulations of specific Parrondo games and mixtures thereof are presented and certain theoretical results are given. These authors also give a heuristic explanation of the paradox in terms of the Brownian ratchet, the original motivation for the suggestion of these games. Other references to the general subject are included in the above mentioned papers by Harmer and Abbott. The reader may also note the reference Durrett, Kesten and Lawler (1991) which also deals with the general question of showing that winning games can be formed by mixing fair ones.

The suggested paradox may be visualized as follows. You are about to play a two-armed slot machine. The casino that owns this two-armed bandit advertises that both arms on their two-armed machines are "fair" in the sense that any player who plays either of the arms is assured that the average cost per play approaches zero as the number of plays increase. However, the casino does not constrain you to stay with one arm; you are allowed to use

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