

# TOETJES NA

THOMAS S. FERGUSON  
*University of California  
at Los Angeles*

CHRISTIAN GENEST  
*Université Laval*

Toetjes is a constant-sum game of perfect information in which players guess in turn the outcome of a random variable  $X$  with known distribution. The contestants, who play in a predetermined order, know the values announced by their predecessors but must make a different choice. The winner is that participant whose guess is closest to the value eventually taken by  $X$ . Feder (*Amer. Math. Monthly*, 1990), who investigated Toetjes as a non-cooperative game when  $X$  is uniformly distributed, observed that it provides a clear advantage to the first guesser, while the last player is at a disadvantage. It is shown here that this result extends to more general densities, but that in contrast, Toetjes is most favorable to the last player when the contestants can form alliances. It is also shown that the last person who speaks has a strong advantage in a version of Toetjes related to the American television show *The Price is Right*, in which the winner is the contestant whose bid is closest to, but less than, the unknown retail price  $X$  of an important collection of consumer goods.

## 1. Introduction

In Dutch, “toetjes” means “afters,” or dessert. In the work of Feder (1990), it also refers to a game that is played around many family dinner tables in the world, at times when a single leftover piece of cake needs to be allocated to one of the children. In the continuous version of the game considered by Feder, a parent secretly picks a number  $X$  at random in the interval  $[0, 1]$  and one by one, in a predetermined order, the  $n \geq 2$  children make their guesses known to all. Child 1 thus selects a number  $x_1$  and, knowing  $x_1, \dots, x_{i-1}$ , Child  $i \geq 2$  chooses  $x_i$  distinct from all previous guesses. Once everybody has spoken, the parent announces whose guess is closest, and this child gets the treat. The others get nothing.

The question addressed by Feder (1990) is that of determining the guess that each child should make in order to maximize his/her probability of winning, knowing that all children do likewise. Feder introduces “tie-breaking” rules to require a specific action when several possible guesses give a child the same maximal probability of win. This paper revisits the issue—“toetjes na” means “after afters” in colloquial Dutch and can be interpreted either as “extra dessert” or “more (on) Toetjes”—in order to characterize the set of optimal strategies from a game-theoretic viewpoint and to explore what happens in situations where the parent’s secret number is drawn from other continuous distributions than the uniform.

Toetjes with  $n = 2$  players is briefly discussed in Section 2, where it is seen to be a game of perfect information that is essentially fair to rational