ACCARDI CONTRA BELL (CUM MUNDI): THE IMPOSSIBLE COUPLING

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An experimentally observed violation of Bell's inequality is supposed to show the failure of local realism to deal with quantum reality. However, finite statistics and the time sequential nature of real experiments still allows a loophole for local realism. We show that the randomised design of the Aspect experiment closes this loophole. Our main tool is van de Geer's (1995, 2000) martingale version of the classical Bernstein (1924) inequality guaranteeing, at the root n scale, a not-heavier-than-Gaussian tail of the distribution of a sum of bounded supermartingale differences. The results are used to specify a protocol for a public bet between the author and L. Accardi, who in recent papers (Accardi and Regoli, 2000a,b, 2001; Accardi, Imafuku and Regoli, 2002) has claimed to have produced a suite of computer programmes, to be run on a network of computers, which will simulate a violation of Bell's inequalites. At a sample size of twenty five thousand, both error probabilities are guaranteed smaller than about one in a million, provided we adhere to the sequential randomized design while Accardi aims for the greatest possible violation allowed by quantum mechanics.

1. Introduction

This paper is concerned with a celebrated paradox of quantum mechanics. Some keywords and phrases are locality, causality, counterfactuals, EPR (Einstein, Podolsky and Rosen, 1935) correlations, the singlet state, entanglement, Bell's (1964) inequalities, and the Aspect experiment (Aspect et al., 1982a,b). However the point of the paper is that almost the whole story can be told in terms of elementary classical probability and statistics. The only physics you should believe, is that the right mathematical model for the periodic, smooth, dependence of a certain correlation coefficient on a certain angle is given by the appropriate sine curve. It seems to me that this little example should be in every probability and statistics course as showing the power of probabilistic reasoning and the importance of statistics in modern day science (it is for instance in Williams, 2001, Chapter 10). Moreover, there is growing realisation that quantum physicists are up to interesting things these days (quantum information, quantum computation, quantum communication), and growing realisation that these things involve probability and

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