TESTING SYMMETRY OF THE ERRORS OF A LINEAR MODEL

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A test for symmetry of the distribution of the errors in a linear model is proposed. It is a goodness-of-fit type test based on the discrepancy between two robust fits. The first fit is appropriate under symmetric errors while the second is appropriate for skewed as well as symmetric distributions. The proposed test is robust and is asymptotically distribution free. Besides deriving the test statistic's null asymptotic distribution, its efficiency under a general class of local alternatives is obtained which allows for the determination of the test's asymptotic relative efficiency with its competitors.

1. Introduction

Constance van Eeden has made many important contributions to the development of signed rank procedures. Symmetry is a crucial assumption necessary for the validity of such procedures. In many situations encountered in practice, a test of symmetry is quite useful. For example, consider a randomized paired design. Under the null hypothesis of no treatment effect, the paired differences are symmetrically distributed; however, under alternatives that involve a change in scale as well as one in location, this is not true.

In this paper, we propose a test for the hypothesis that the errors in a linear model are symmetrically distributed. It is a goodness-of-fit type test based on the discrepancy between two robust rank-based fits. The first fit uses a robust signed-rank (SR) fitting criterion that is appropriate under the assumption of symmetric errors. It yields the distance, the minimum of the objective function, $D_{\text{SR}}(\widehat{\mathbf{Y}}_{\text{SR}})$, between the vector of responses, $\mathbf{Y}$, and the vector of fitted values, $\widehat{\mathbf{Y}}_{\text{SR}}$. The second fitted vector, $\widehat{\mathbf{Y}}_{\text{R}}$, is based on a robust rank (R) fitting criterion that is appropriate for either symmetric or asymmetric error distributions. For this second fit we obtain $D_{\text{SR}}(\widehat{\mathbf{Y}}_{\text{R}})$ the distance between $\mathbf{Y}$ and $\widehat{\mathbf{Y}}_{\text{R}}$ using the symmetric "yardstick," i.e., distance based on the SR norm. The test statistic is the standardized difference in these distances, $H_\varphi = RD/\widehat{\delta}$ where $RD = D_{\text{SR}}(\widehat{\mathbf{Y}}_{\text{R}}) - D_{\text{SR}}(\widehat{\mathbf{Y}}_{\text{SR}})$; see (2.13). Under symmetry, the fits, and hence the distances, should be similar. Thus the null hypothesis of symmetry is rejected for large values of RD.

Keywords and phrases: asymptotic distribution-free; asymptotic relative efficiency; linear rank scores; rank-based regression; robust; signed-rank regression; Wilcoxon scores.