ALIGNED RANK TEST FOR THE BIVARIATE RANDOMIZED BLOCK MODEL

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An aligned rank test for treatment effects in the bivariate randomized block model is proposed. The test is easy to implement and its validity requires only minimal assumptions. Furthermore, the test statistic is affine-invariant and has a limiting χ^2 distribution under the null hypothesis when the number of blocks goes to infinity. If the number of blocks is not large enough, we show how to perform a permutation test and illustrate this method with an example. Finally, a simulation study indicates that the new test performs well compared to the likelihood ratio test, to a coordinate-wise aligned rank test and to a sign test based on the Oja measure of scatter.

1. Introduction

Consider the bivariate randomized block model with one observation in each cell. We wish to test the hypothesis that there are no treatment effects. If we assume that the observations follow a bivariate normal distribution, one sensible test is based on the likelihood ratio statistic described, for example, in Section 8.9 of Anderson (1984). This statistic is affine-invariant, that is, its value remains unchanged if a nonsingular linear transformation is applied to the observations. In practice, this important property means that the power of the test is not affected by the correlation structure or the scale of the variables.

For univariate data, there are two main approaches to construct tests based on ranks. The first one uses intra-block ranking which means that the observations are ranked separately within each block. The Friedman test is the most well-known example of this approach, Hollander and Wolfe (1999). This test is distribution-free but its efficiency relative to the classical variance-ratio test is quite low at the normal model when the number of treatment is small and this is essentially due to the fact that no inter-block comparisons are made. To alleviate this problem, Quade (1979) proposed a method that is still based on within-block ranking but where each block is given a data-driven weight. His test does have a better efficiency when the number of treatments is small and remains distribution-free. On the other hand, as opposed to the Friedman test, its efficiency decreases as the number of treatments increases, see Table 1 of Tardif (1987). Larocque

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