ASYMPTOTICALLY MOST ACCURATE CONFIDENCE INTERVALS IN THE SEMIPARAMETRIC SYMMETRIC LOCATION MODEL

CHRIS A.J. KLAASSEN
University of Amsterdam

One- and two-sided confidence intervals are considered for the location parameter in the semiparametric symmetric location model. Asymptotic bounds are proved and confidence intervals are constructed that attain these bounds locally asymptotically uniformly. Global uniformity is studied as well.

1. Introduction

Let $\mathcal{G}$ be the class of distribution functions G with densities g w.r.t. Lebesgue measure that are symmetric about 0 and that have finite Fisher information $I(G)$ for location. This means that every $G \in \mathcal{G}$ has a density g satisfying

$$g(-x) = g(x), \quad x \in \mathbb{R}, \tag{1.1}$$

and being absolutely continuous with derivative g' such that

$$I(G) = \int (g'/g)^2 g < \infty \tag{1.2}$$

holds. The semiparametric symmetric location model

$$\mathcal{P} = \{P_{\theta,G} : \theta \in \mathbb{R}, G \in \mathcal{G}\} \tag{1.3}$$

consists of all distributions $P_{\theta,G}$ with density $g(x-\theta)$, $x \in \mathbb{R}$, with respect to Lebesgue measure.

Based on i.i.d. random variables $X_1, \dots, X_n$ with distribution $P_{\theta,G}$ estimation of the location parameter θ is possible by estimator sequences $(T_n)_{n\in\mathbb{N}} = \big(t_n(X_1,\dots,X_n)\big)_{n\in\mathbb{N}}$ satisfying

$$\sqrt{n}(T_n - \theta) \xrightarrow{\mathcal{D}} \mathcal{N}\big(0, I^{-1}(G)\big), \quad \theta \in \mathbb{R}, G \in \mathcal{G}, \tag{1.4}$$

and even

$$\sqrt{n}\left(T_n - \theta + \frac{1}{n}\sum_{i=1}^{n} I^{-1}(G)\frac{g'}{g}(X_i - \theta)\right) \xrightarrow{P_{\theta,G}} 0, \tag{1.5}$$

AMS subject classifications: Primary 62G15; Secondary 62G05, 62G20.
Keywords and phrases: Adaptive estimation, local uniformity, global uniformity, asymptotic bounds.