SOME ASPECTS OF MATCHING PRIORS

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Priors for which Bayesian and frequentist inference agree, at least to some order of approximation, are called 'matching priors', and have been proposed as candidates for noninformative priors in Bayesian inference. We give an overview of the original work of Welch and Peers and some more recent developments.

1. Introduction

In the context of parametric inference, a matching prior is a prior for which posterior probability statements about the parameter also have an interpretation as confidence statements in the sampling model. The idea appears to have been proposed first by Lindley (1958). There have been several attempts to develop matching priors, starting with Welch and Peers (1963). Matching priors in principle hold the promise of providing a possible frequentist/Bayesian compromise and of providing default priors for routine use in Bayesian inference. This is attractive from some frequentist points of view because the Bayesian approach to inference provides a simple way to eliminate nuisance parameters, and typical frequentist approaches are rather more complicated. Default priors are attractive from some Bayesian points of view as they might be expected to be more widely accepted than subjective priors. In addition the inference from a default prior can be compared to that from priors developed otherwise, as a possible check on the robustness of the inference to the prior.

We consider here priors that lead to approximate matching, to some order of approximation in the sample size n. The terminology in the literature is not standardized, and we follow here the version used in Mukerjee and Reid (1999): we call a prior *first order matching* if it ensures approximate frequentist validity of a Bayesian posterior credible set with margin of error $O(n^{-1})$, and *second order matching* if it does so with a margin of error of $O(n^{-3/2})$. Since the posterior distribution is typically asymptotically normal for any choice of prior, it is only at the $O(n^{-1/2})$ term of the asymptotic expansion that matching leads to a class of priors. The relevant asymptotic expansions are typically in powers of $n^{-1/2}$, so first order matching actually involves the second order term in the expansion.

Welch and Peers (1963) showed that Jeffreys' prior is the unique first order matching prior in sampling from a model with a scalar parameter.