OLD AND NEW ASPECTS OF MINIMAX ESTIMATION OF A BOUNDED PARAMETER

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In practice the unknown parameter of an experiment is often bounded. Therefore there is practical interest to include such additional knowledge into statistical procedures to improve them. In this paper we consider minimax estimation of a bounded parameter. The minimax principle is, at least from a theoretical point of view, very important. In the last two decades of the last century a sequence of papers has treated such problems for some densities that are smooth with respect to the unknown parameter and for specific convex losses. It occurred that in all examples in which the loss is strictly convex and the parameter interval is sufficiently small a minimax estimator exists which is Bayes with respect to a two-point prior with mass at the boundaries. In this paper we show that this result is true in general when the Lebesgue-densities are smooth with respect to the unknown parameter and the loss is strictly convex, but do not necessarily penalize equally underestimation and overestimation. Our result provides new classes of interesting losses and densities for which the above statement holds true.

1. Introduction

In practice there are typical situations where it is most appealing to apply the minimax principle. On the other hand, there are several reasons refraining the utilization of minimax rules, for example:

- minimax rules are difficult to calculate,
- there is no universal method to construct a minimax rule,
- examples are known for which a decision rule exists whose maximal risk is a little larger than the minimax risk but whose risk function is much better than the risk function of the minimax rule for large parts of the parameter space. This is especially the case when the minimax risk function is constant. In such a situation the minimax rule is not attractive to a practitioner.

Nevertheless, from a theoretical point of view it is important to obtain minimax rules, see Brown (1994). Note that only if the risk function of the minimax rule is known, rules that are more convenient with respect to the risk function can be developed. Moreover, the situation is a little different in case where the parameter space is restricted because then the risk function of the minimax estimator is typically not constant. Consequently the risk functions of minimax estimators for restricted problems are often quite more appealing than in the unbounded case where they are often constant.