ON MINIMAX ESTIMATION OF A NORMAL MEAN VECTOR FOR GENERAL QUADRATIC LOSS

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Let $X \sim N_p(\theta, \Sigma)$ (Σ known) and consider the problem of estimating the mean vector when loss is general quadratic loss $(\delta - \theta)'Q(\delta - \theta)$. Many results are known for the case $\Sigma = Q = I$. There is also a relatively large literature for the case of general Σ and Q but it is relatively less well developed. The purpose of this paper is to unify many of the results in the general case by relating them to the simpler case $\Sigma = Q = I$. We give a reduction of the general case to a canonical form ($\Sigma = I$, $Q =$ Diagonal) and show that a natural correspondence between priors, marginals, and estimators in the two versions of the problem preserves risk, admissibility, minimaxity and Bayesianity. This allows many results on minimaxity and admissibility in the case $\Sigma = Q = I$ to be extended to the general case and allows an expansion of the classes of known minimax estimators in the general case. It also seems to make the general case somewhat more comprehensible.

1. Introduction

Let $X \sim N_p(\theta, \Sigma)$ and consider the problem of estimating the mean vector θ with loss $L(\theta, d) = (d - \theta)'Q(d - \theta)$.

A great deal is known about this problem when $\Sigma = Q = I$ (and more generally when Σ and Q are known multiples of I). Relatively less is known when the covariance matrix, Σ, and the matrix Q are general positive definite matrices. The purpose of this paper is to close, to a degree, the gap between the case $Q = \Sigma = I$ and the general case.

In Section 2, we briefly present a snapshot of results for the case where Σ and Q are known multiples of I. In Section 3, we extend these results to the case where Σ and Q are diagonal and in Section 4, to the case of general positive definite Σ and Q.

The spirit of the development herein is to derive procedures in the general case corresponding to procedures in the $\Sigma = Q = I$ case which are Bayes (proper, generalized, or pseudo) minimax and/or admissible and which preserve these properties in the general case. There are a number of results along these lines in the literature. This paper unifies and generalizes many of these results and gives a comprehensive and, it is hoped, comprehensible picture of the general case.

We will use the notation $\nabla m(X)$, $\nabla \cdot m(X)$ and $\nabla^2 m(X)$ for the gradient, divergence, and laplacian of a function $m(X)$. Recall that $\nabla^2 m(X) = \sum_1^p \partial^2 m(X)/\partial X_i^2$ and that a function $m(X)$ is superharmonic if and only if $\nabla^2 m(X) \leq 0 \; \forall X$.

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