A Theorem of Large Deviations for the Equilibrium Prices in Random Exchange Economies

Esa Nummelin
University of Helsinki

Abstract

We formulate and prove a theorem concerning the large deviations of equilibrium prices in large random exchange economies.

1 Introduction

We consider an *economic system* (shortly, *economy*) $\mathcal{E}$, where certain *commodities* $j = 1, ..., l$ are traded. Let $R^l_+ =_{\text{def}} \{p = (p^1, ..., p^l) \in R^l;\ p^j \geq 0 \text{ for all } j = 1, ..., l\}$. The elements p of R^l_+ are interpreted as *price vectors* (shortly, *prices*). (We will follow a convention, according to which superscripts always refer to the commodities whereas subscripts refer to the economic agents.)

The *total excess demand function* $Z(p) = (Z^1(p), ..., Z^l(p)) \in R^l$ comprises the total excess demands on the l commodities in the economy at the prices $p \in R^l_+$. Its zeros p^* are called the *equilibrium prices*:

$$Z(p^*) = 0.$$

(In fact, according to Walras' law, we may regard money as an $l+1$'st commodity [the numeraire] having price $p^{l+1} = 1$ and total excess demand $Z^{l+1}(p) = -p \cdot Z(p)$.)

In the classical equilibrium theory the economic variables and quantities are supposed to be deterministic, see [2]. It is, however, realistic to allow uncertainty in an economic model.

We assume throughout this paper that the total excess demand $Z(p)$ is a random variable (for each fixed price p). In particular, it then follows that the equilibrium prices p^* form a *random set.*

The seminal works concerning equilibria of random economies are due to Hildenbrand [5], Bhattacharya and Majumdar [1] and Föllmer [4].

The equilibrium prices in large random economic systems obey (under appropriate regularity conditions) classical statistical limit laws.

The *law of large numbers* [1] states that, as the number n of economic agents increases, the random equilibrium prices (r.e.p.'s) p^*_n become asymptotically equal to deterministic "expected" equilibrium prices:

$$\lim_{n\to\infty} p^*_n = p^*_e.$$