Polynomially Harmonizable Processes and Finitely Polynomially Determined Lévy Processes

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Abstract

The sequence $\{P_k(t,x)\}$ of two-variable Hermite polynomials are known to have the property that, if $\{M_t, t \geq 0\}$ denotes the standard Brownian motion, then $P_k(t, M_t)$ is a martingale for each $k \geq 1$. This property of standard Brownian motion vis-a-vis Hermite polynomials motivated the general notion of "polynomially harmonizable processes". These are processes that admit sequences of time-space harmonic polynomials, that is, two-variable polynomials which become martingales when evaluated along the trajectory of the process. For Lévy processes, this property is connected to certain properties of the associated Lévy/Kolmogorov measures. Moreover, stochastic properties of the underlying processes (like independence, stationarity of increments) turn out to be equivalent to certain algebraic/analytic properties of the corresponding sequence of polynomials. We first present a brief survey of these recently obtained general results and then describe necessary and sufficient conditions for certain classes of Lévy processes to be uniquely determined by a finite number of time-space harmonic polynomials.

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1 Introduction: General Definitions

The sequence of two-variable Hermite polynomials $\{P_k,\ k \geq 1\}$ on $[0,\infty) \times \mathbb{R}$ are defined via the classical one-variable Hermite polynomials $\{p_k,\ k \geq 1\}$ as follows:

$$P_k(t,x) = t^{k/2} p_k(\frac{x}{\sqrt{t}}),$$

where

$$p_k(x) = (-1)^k e^{x^2/2} \frac{\partial^k}{\partial x^k}(e^{-x^2/2}).$$

Some of the well-known properties of the sequence $\{P_k\}$ are:

- $P_k(t,x)$ is a polynomial in the two variables t and x, for each k.