On Conditional Central Limit Theorems For Stationary Processes

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Abstract

The central limit theorem for stationary processes arising from measure preserving dynamical systems has been reduced in [6] and [7] to the central limit theorem of martingale difference sequences. In the present note we discuss the same problem for conditional central limit theorems, in particular for Markov chains and immersed filtrations.

1 Introduction

Let $(\zeta_k)_{k\in\mathbb{Z}} = ((\xi_k, \eta_k))_{k\in\mathbb{Z}}$ be a two-component strictly stationary random process. Every measurable real-valued function f on the state space of the process defines another stationary sequence $(f(\zeta_k))_{k\in\mathbb{Z}}$. Various questions in stochastic control theory, modeling of random environment among many other applications lead to the study of conditional distributions of the sums $\sum_{k=0}^{n-1} f(\zeta_k)$ given $\eta_0, \ldots, \eta_{n-1}$. In particular, the asymptotic behaviour of these conditional distributions is of interest, including the case when the limit distribution is normal.

We shall prove conditional central limit theorems in the slightly more abstract situation of measure preserving dynamical systems $(X, \mathcal{F}, P, T)$, where $(X, \mathcal{F}, P)$ is a probability space and $T : X \to X$ is P-preserving.

Let f be a measurable function and $\mathcal{H}$ be a sub-σ-algebra. f is said to satisfy the conditional central limit theorem with respect to $\mathcal{H}$ (CCLT($\mathcal{H}$)), if P a.s. the conditional distributions of

$$\frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} f \circ T^k,$$

given $\mathcal{H}$, converge weakly to a normal distribution with some non-random variance $\sigma^2 \geq 0$.

This leads to the identification problem for $L_2(P)$-subspaces consisting of functions satisfying a CCLT. Following [6], an elegant way to describe such subclasses

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