Transition Density of a Reflected Symmetric Stable Lévy Process in an Orthant

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Abstract

Let $\{Z^{(s,x)}(t) : t \geq s\}$ denote the reflected symmetric α-stable Lévy process in an orthant D (with nonconstant reflection field), starting at (s, x). For $1 < \alpha < 2, 0 \leq s < t, x \in \overline{D}$ it is shown that $Z^{(s,x)}(t)$ has a probability density function which is continuous away from the boundary, and a representation given.

1 Introduction

Due to their applications in diverse fields, symmetric stable Lévy processes have been studied recently by several authors; see [4], [5] and the references therein. In the meantime reflected Lévy processes have been advocated as heavy traffic models for certain queueing/stochastic networks; see [14]. The natural way of defining a reflected/regulated Lévy process is via the Skorokhod problem as in [9], [3], [11], [1].

In this article we consider reflected/regulated symmetric α-stable Lévy process in an orthant, show that transition probability density function exists when $1 < \alpha < 2$ and is continuous away from the boundary; the reflection field can have fairly general time-space dependencies as in [11]. It may be emphasized that unlike the case of reflected diffusions (see [10]) powerful tools/methods of PDE theory are not available to us. To achieve our purpose we use an analogue of a representation for transition density (of a reflected diffusion) given in [2].

Section 2 concerns preliminary results on symmetric α-stable Lévy process in $\mathbb{R}^d$, its transition probability density function and the potential operator. In Section 3, corresponding reflected process with time-space dependent reflection field at the boundary is studied. A major effort goes into proving that the distribution of the reflected process at any given time $t > 0$ gives zero probability to the boundary.

2 Symmetric stable Lévy process

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ be a filtered probability space, $d \geq 2, 0 < \alpha < 2$. Let $\{B(t) : t \geq 0\}$ be an $\mathcal{F}_t$-adapted d-dimensional *symmetric α-stable Lévy process*. That is, $\{B(t)\}$ is an $\mathbb{R}^d$-valued homogeneous Lévy process (with independent increments) with r.c.l.l. sample paths; it is roation invariant and

$$E[\exp\{i\langle u, B(t) - x\rangle\}|B(0) = x] = \exp\{-t|u|^\alpha\} \tag{2.1}$$