Simulating Constrained Animal Motion Using Stochastic Differential Equations

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Abstract

Differential equations have long been used to describe the motion of particles. Stochastic differential equations (SDE)s have been employed for situations where randomness is included. This present work is motivated in part by seeking to describe the motion of mammals moving in a constrained region. Interesting questions that arise include: how to write down a pertinent (bivariate) SDE, how to include explanatories, and boundaries and how to simulate realizations of a process?

1 Introduction

Differential equations have long been used to describe the motion of particles and stochastic differential equations (SDE)s have been employed for situations where there is randomness. Our work is motivated in part by the case of ringed-seals, elephant seals, cows, elk and deer. The last three are moving about together in an experimental forest in Oregon.

The study was influenced by emerging data sets in wildlife biology. Biologists and managers wish to use these data sets to address questions such as: how to allocate resources, can different species share a habitat, are changes taking place? One large experiment, Starkey, is described in [6] and [23].

There are technical questions arising of interest to both probabilists and statisticians. Useful tools include: differential equations (DE)s, stochastic differential equations (SDE)s, reflecting stochastic differential equations (RSDE)s, and potential functions

The paper includes review and the results of some elementary simulations, particularly for the case of constrained motion having in mind future data analyses. The work is preparatory to employing simulated realizations of SDE models.

The sections of the paper are: Introduction, Some wildlife examples, Equations of motion, Stochastic differental equations, The constrained case, Results of some simulations, Several particles, and Discussion.

2 Some wildlife examples

The work of the paper, particularly the need to consider bounded domains, may be motivated by some examples from wildlife biology.

Figure 1 shows the motion of a ringed seal as recorded in the Barrow Strait, North West Territories. The animal is constrained within an ice-covered lake that has several air holes. The trip starts at the dot. The animal dives to the bottom, swims around then returns to the air hole. It also looks at another air hole. The locations are available at irregular time intervals. The researchers were concerned with the animal's navigation, foraging and use of its underwater habitat. To study its navigational sense the eyes of the animal were covered during the dive graphed. The ecology of ringed seals is described in [14].