Adaptive Estimation of Directional Trend

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Abstract

Consider a one-way layout with one directional observation per factor level. Each observed direction is a unit vector in R^p measured with random error. Information accompanying the measurements suggests that the mean directions, normalized to unit length, follow a trend: the factor levels are ordinal and mean directions at nearby factor levels may be close. Measured positions of the paleomagnetic north pole in time illustrate this design. The directional trend estimators studied in this paper stem from penalized least squares (PLS) fits in which the penalty function is the squared norm of first-order or second-order differences of mean vectors at adjacent factor levels. Expressed in spectral form, such PLS estimators suggest a much larger class of monotone shrinkage estimators that use the orthogonal basis implicit in PLS. Penalty weights and, more generally, monotone shrinkage factors are selected to minimize estimated risk. The possibly large risk reduction achieved by such adaptive monotone shrinkage estimators reflects the economy of the PLS orthogonal basis in representing the actual trend and the flexibility of unconstrained monotone shrinkage.

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1 Introduction

Consider n independent measurements taken successively in time on the varying position of the earth's north magnetic pole. Each measured position may be represented as a unit vector in R^3 that gives direction from the center of the earth to the north magnetic pole. Because of measurement errors, it is plausible to model the data as a realization of independent random unit vectors $\{y_i : 1 \leq i \leq n\}$ whose mean vectors are $\eta_i = \mathrm{E}(y_i)$. The subscript i labels time. The *mean direction* of y_i is defined to be the unit vector $\mu_i = \eta_i/|\eta_i|$. In this example, the mean directions $\{\mu_i\}$ follow a trend, by which we mean that the subscript order matters and that the distance between μ_i and μ_j may be relatively small when i is close to j.

The naive estimator of μ_i is $\hat{\mu}_{N,i} = y_i$. It can be derived as the maximum likelihood estimator of μ_i when the distribution of y_i is Fisher-Langevin with mean direction μ_i and precision parameter κ. Unless measurement error is very small, $\{\hat{\mu}_{N,i}\}$ is not a satisfactory estimator of the directional trend $\{\mu_i\}$. If we foresee that the trend in the means $\{\eta_i\}$ may possess some degree of smoothness, not known to us, it is natural to look for more efficient estimators within classes of smoothers. In an instructive data analysis, Irving (1977) suggested forming local symmetric weighted averages of the $\{y_i\}$, normalizing these to unit length so as to obtain a more revealing estimator of directional trend.

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