Designs on Association Schemes

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Abstract

An *association scheme* partitions a finite set Ω into symmetric subsets, one of which is the diagonal subset. This paper develops the idea of a design map between two association schemes. In many designed experiments, the structure on the experimental units is an orthogonal block structure. These appear to be the structures where both the components-of-variance and patterns-of-covariance approaches (almost) agree. By replacing orthogonal block structures by association schemes, only the patterns-of-covariance model generalizes.

Keywords: association schemes; balanced design; experimental design; general balance; Latin square; orthogonal block structures

1 Introduction

Terry Speed and I worked together in the 1980s on problems in the analysis of variance. My motivation was to understand how an analysis of variance could be defined by the randomization used in setting up the experiment [3]; his was more fundamental, seeking to answer the question 'What is an analysis of variance?' [29]. We were both heavily influenced by John Nelder's two papers [25, 26], in which he defines simple orthogonal block structures, makes an unsubstantiated claim about randomization, defines general balance, and shows how to analyse data from generally balanced experiments with many strata.

In joint work with Cheryl Praeger and Chris Rowley [7], we were able to generalize Nelder's simple orthogonal block structures to a class which I now call poset block structures, and prove that Nelder's claim about randomization holds in poset block structures. The other three authors extended this work in [27], while I showed in [4] that poset block structures are the same as the 'complete balanced response structures' which Kempthorne and his team at Ames, Iowa had studied extensively [21, 22, 32, 36].

More surprisingly, in [30, 31] Speed and I found that if you ignore the question of randomization then you can define an even wider class of structures in which all of Nelder's theory carries through, with rather easy proofs. Today I use the term 'orthogonal block structure' for structures in this class [4]. An important input from Speed was to recognise that these orthogonal block structures are association schemes: this insight has influenced my own subsequent work enormously. A second key input from Speed was to introduce concepts from partial orders, most importantly the Möbius function,