Investigating the Structure of Truncated Lévy-stable Laws

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Abstract

Truncations of stable laws have been proposed in the econophysics literature for modelling financial returns, often with imprecise definitions. This paper sharpens definitions of exponential truncations and attempts to expose underlying structure. Analytical comparisons are made with alternative models, leading to a tentative conclusion that the generalized hyperbolic family is more attractive for empirical work.

Keywords: Truncation; Lévy-stable laws

1 Introduction

Extensive empirical research shows that (log-)return data obtained from frequently sampled financial time series is not well fitted by a normal (Gaussian) law. Rather, the 'true' population law is more peaked around its median and it has fatter tails. Many analytically specified laws have been proposed and found to give a good fit to selected data sets. For example McDonald [22], Rydberg [31], and Voit [35, §5.3,5.4] are recent reviews representing the finance, statistics, and physics disciplines, respectively. In particular, Mandelbrot [23, E14,15] champions validity of non-normal stable laws. In fact, many return series exhibit tail behaviour which is intermediate to normal and non-normal stable behaviour. As a result, various more complicated models built from stable laws are found to mimic the stylized features of real data; see [31, 3].

The 'econophysics' school of modellers support use of so-called truncated Lévy (*i.e.*, stable) laws. See [7] for a general discussion of their use in finance, and [25] for pricing options. Let $g(x;\alpha)$ denote the density function of a stable law having index $\alpha \in (0,2)$ and symmetric about the origin, and let X be a random variable having this law. If $1 < \alpha < 2$ then $E(X) = 0$ and $var(X) = \infty$, but if $0 < \alpha \leq 1$ then neither the mean nor the variance can be defined. Econophysicists hold this to be unsatisfactory on the reasonable grounds that returns cannot be arbitrarily large in magnitude, and hence admissible models should possess finite moments of all orders. In general terms, the solution they propose is to use weighted densities

$$f(x;\alpha,w) = w(x)g(x;\alpha), \tag{1.1}$$