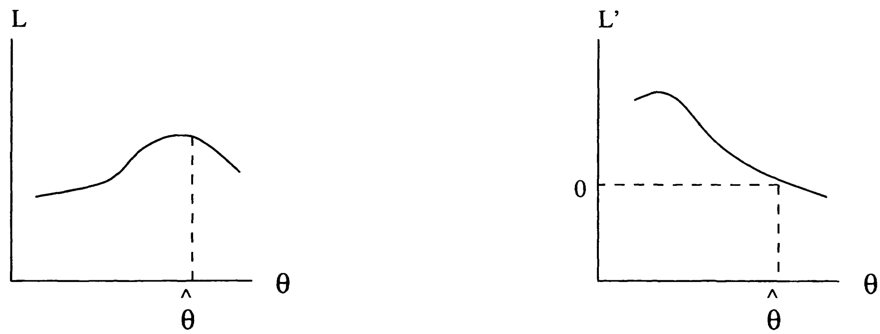
Chapter 7

Lecture 25

Using the score function (or vector)

Assume the usual setting, $(S, \mathcal{A}, P_\theta)$, $\theta \in \Theta \subseteq \mathbb{R}^p$.



First consider the case $p = 1$. Let $u(s)$ be a trial solution of $L'(\theta \mid s) = 0$. Assume that $\hat{\theta} - \theta = O(1/\sqrt{I(\theta)})$ and I is large. (Here θ is the true parameter, $E_\theta(\hat{\theta}) \approx 0$ and $\mathrm{Var}_\theta(\hat{\theta}) \approx 1/I(\theta)$.) Assume that u is not very inaccurate in the sense that, for any θ, $u(s) - \theta = O(1/\sqrt{I(\theta)})$. Then $\hat{\theta} - u = O(1/\sqrt{I(\theta)})$ under θ,

$$0 = L'\big(\hat{\theta}(s) \mid s\big) = L'(u(s) \mid s) + \big(\hat{\theta}(s) - u(s)\big)L''(u(s) \mid s) + O(1/I(\theta))$$

and

$$\hat{\theta}(s) = u(s) + \left(-\frac{1}{L''(u(s) \mid s)}\right)L'(u(s) \mid s) + O(1/I(\theta)).$$

Dropping the last term (order $1/I(\theta)$), we obtain the 'first Newton iterate' for solving $L'(\theta \mid s) = 0$.

Application 1. Let $u^{(0)}(s)$ be a trial solution of $L'(\theta \mid s) = 0$. Let

$$u^{(j+1)}(s) = u^{(j)}(s) + \left(-\frac{1}{L''(u^{(j)}(s) \mid s)}\right)L'(u^{(j)}(s) \mid s).$$

One hopes that $u^{(j)}(s) \to \hat{\theta}(s)$.