Chapter 6

Lecture 19

The vector-valued score function and information in the multi-parameter case

Now we have an experiment $(S, \mathcal{A}, P_\theta)$, $\theta = (\theta_1, \ldots, \theta_p) \in \Theta$ with Θ an open set in $\mathbb{R}^p$ and a smooth function $g : \Theta \to \mathbb{R}^1$. We assume that $dP_\theta(s) = \ell_\theta(s) d\mu(s)$ as before, and define $\ell(\theta \mid s) := \ell_\theta(s)$. Assume that ℓ is smooth in θ and let $g_i(\theta) = \frac{\partial}{\partial \theta_i} g(\theta)$, $\ell_i(\theta \mid s) = \frac{\partial}{\partial \theta_i} \ell(\theta \mid s)$ and $\ell_{ij}(\theta \mid s) = \frac{\partial^2}{\partial \theta_i \partial \theta_j} \ell(\theta \mid s)$ for $1 \leq i, j \leq p$. There are two approaches to the present topic in this situation:

Approach 1. Generalize the previous one-dimensional discussion: Suppose that t is unbiased for g – that is to say,

$$\int_S t(s) \ell(\delta \mid s) d\mu(s) = E_\delta(t) = g(\delta)$$

for all $\delta \in \Theta$. Then

$$E_\theta\big(t(s)\ell_i(\theta \mid s)/\ell(\theta \mid s)\big) = \int_S t(s)\ell_i(\theta \mid s) d\mu(s) = g_i(\theta)$$

for $i = 1, \ldots, p$ and hence every $t \in U_g$ has the same projection on $\operatorname{Span}\{1, L_1, \ldots, L_p\}$, where $L(\theta \mid s) = L_\theta(s)$ and

$$L_i(\theta \mid s) = \frac{\partial}{\partial \theta_i} L(\theta \mid s) = \frac{\ell_i(\theta \mid s)}{\ell(\theta \mid s)}.$$

This approach is useful for studies of conditions which ensure that $L_1, L_2, \ldots, L_p$ are in $W_\theta = \operatorname{Span}\{\Omega_{\delta,\theta} : \delta \in \Theta\}$.

Approach 2. Use the result for the θ-real case: Fix $\theta \in \Theta$ and a vector $c = (c_1, \ldots, c_p) \neq 0$, and suppose that δ is restricted to the line passing through θ and $\theta + c$ – in other words, that we consider only $\delta = \theta + \xi c$ for some scalar ξ. (Note that, since Θ is