Chapter 1

Note on the notation: Throughout, Professor Bahadur used the symbols $\varphi(s)$, $\varphi_1(s)$, $\varphi_2(s)$, ... to denote functions of the sample that are generally of little importance in the discussion of the likelihood. These functions often arise in his derivations without prior definition.

Lecture 1

Review of L^2 geometry

Let $(S, \mathcal{A}, P)$ be a probability space. We call two functions f_1 and f_2 on S EQUIVALENT if and only if $P(f_1 = f_2) = 1$, and set

$$V = L^2(S, \mathcal{A}, P) := \{f : f \text{ is measurable and } E(f^2) = \int_S f(s)^2 dP(s) < \infty\},$$

where we have identified equivalent functions. We may abbreviate $L^2(S, \mathcal{A}, P)$ to $L^2(P)$ or, if the probability space is understood, to just L^2. For $f, g \in V$, we define $||f|| = +\sqrt{E(f^2)}$ and $(f, g) = E(f \cdot g)$, so that $||f||^2 = (f, f)$. Throughout this list f and g denote arbitrary (collections of equivalent) functions in V.

1. V is a real vector space.

2. $(\cdot, \cdot)$ is an inner product on V – i.e., a bilinear, symmetric and positive definite function.

3. CAUCHY-SCHWARZ INEQUALITY:

$$|(f, g)| \leq ||f|| \cdot ||g||,$$

with equality if and only if f and g are linearly dependent.

Proof. Let x and y be real; then, by expanding $||\cdot||$ in terms of $(\cdot, \cdot)$, we find that

$$0 \leq ||xf + yg||^2 = x^2||f||^2 + 2xy(f, g) + y^2||g||^2,$$

from which the result follows immediately on letting $x = ||g||$ and $y = ||f||$. □