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OPTIMAL ESTIMATING EQUATIONS FOR MIXED EFFECTS MODELS WITH DEPENDENT OBSERVATIONS

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Abstract

Optimal joint estimating equations for fixed and random parameters are derived via an extension of the Godambe criterion. Applications to autoregressive processes and generalized linear mixed models for Markov processes are discussed. Marginal optimal estimating functions for fixed parameters are also discussed.
Key Words: Optimal Estimating Functions; Generalized Linear Mixed Models; Autoregressive Processes; Markov Processes.

1 Introduction

Mixed effects models containing both fixed and random parameters are used extensively in both applied and methodological literature. A review of linear mixed models and their applications is given by Robinson(1991). Generalized linear mixed models are discussed by Breslow and Clayton(1993) among others. Work on the extension of mixed effects models to dependent observations appears relatively scarce. Our main goal in this paper is to develop optimal estimating equations for mixed effects models with dependent observations. See Basawa et al.(1997), Heyde(1997), and Godambe(1991) for recent literature on optimal estimating functions. Desmond(1997) gives an overview of estimating functions.

Suppose Y_t is a vector of observations on n individuals at time t, and $Y(t-1) = (Y_1, \ldots, Y_{t-1})$. Conditional on $Y(t-1)$ and a random parameter γ, the density of Y_t is denoted by $p(y_t|y(t-1), \beta, \gamma)$ where β is a fixed parameter. Let $\pi(\gamma|\alpha)$ denote the (prior) density of γ which may depend on a parameter α. Suppose, for simplicity, α is known, and we wish to estimate β and γ from a sample $Y(T) = (Y_1, \ldots, Y_T)$.

The likelihood function, conditional on γ, is given by

$$L(\beta, \gamma) = p(y_0)\Pi_{t=1}^{T} p(y_t|y(t-1), \beta, \gamma). \tag{1.1}$$