PLUG-IN ESTIMATORS
IN SEMIPARAMETRIC STOCHASTIC PROCESS MODELS

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Abstract

Consider a locally asymptotically normal semiparametric model with a real parameter ϑ and a possibly infinite-dimensional parameter F. We are interested in estimating a real-valued functional $a(F)$. If $\hat{a}_\vartheta$ estimates $a(F)$ for known ϑ, and $\hat{\vartheta}$ estimates ϑ, then the plug-in estimator $\hat{a}_{\hat{\vartheta}}$ estimates $a(F)$ if ϑ is unknown. We show that $\hat{a}_{\hat{\vartheta}}$ is asymptotically linear and regular if $\hat{a}_\vartheta$ and $\hat{\vartheta}$ are, and calculate the influence function and the asymptotic variance of $\hat{a}_{\hat{\vartheta}}$. If $a(F)$ can be estimated adaptively with respect to ϑ, then $\hat{a}_{\hat{\vartheta}}$ is efficient if $\hat{a}_\vartheta$ is efficient. If $a(F)$ cannot be estimated adaptively, then for $\hat{a}_{\hat{\vartheta}}$ to be efficient, $\hat{\vartheta}$ must also be efficient. We illustrate the results with stochastic process models, in particular with time series models, and discuss extensions of the results.

Key Words: Empirical estimator, asymptotically linear estimator, influence function, regular estimator, Markov chain model, nonlinear regression, residual distribution, nonlinear autoregression, innovation distribution, stochastic equicontinuity, stochastic differentiability.

1 Introduction

Let $\overline{\mathcal{P}}_n = \{P_{n\vartheta F} : \vartheta \in \Theta, F \in \mathcal{F}\}$ denote a sequence of semiparametric models, with Θ one-dimensional and $\mathcal{F}$ a possibly infinite-dimensional set. We are interested in estimating a real-valued functional $a(F)$. For each ϑ