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STABILITY OF NONLINEAR TIME SERIES: WHAT DOES NOISE HAVE TO DO WITH IT?

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Abstract

We survey results on the stability of various nonlinear time series, both parametric and nonparametric. The emphasis will be on identifying the role that the "error term" has in determining stability. The error term can indeed affect stability, even when additive and for simple, common parametric models. The stability of the time series is not necessarily the same as that of its related (noiseless) dynamical system. In particular, this means that care must be taken to ensure that estimates are actually within the valid parameter space when analyzing a nonlinear time series.

Key Words: ergodicity, Markov chain, nonlinear time series

1 Introduction

Fitting time series with nonlinear models has become increasingly popular, especially since the emergence of nonparametric function estimation methods (Collomb and Härdle (1986), Härdle and Vieu (1992), Chen and Tsay (1993a,b), Tjøstheim and Auestad (1994a,b), Masry and Tjøstheim (1995)). No matter what model is fit, a critical part of the estimation procedure is determining whether the model is stable or whether the parameters are within the appropriate parameter space (Tjøstheim (1994)). Additionally, knowing the stability properties of a particular model makes it possible to develop simulation and resampling procedures to be used for inference.

For these procedures, as well as for the obvious questions of limit theorems and robustness, the nature of the noise (error) distribution is clearly a significant concern. What is not so clear, however, is how this distribution can affect — if it does at all — the stability question itself. Habit with