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Estimation of the Long-memory Parameter: a Review of Recent Developments and an Extension

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Abstract

Current methods of estimating the memory parameter, d, of a long-range dependent stationary time series are reviewed, and a new method of estimating d by fitting a fractionally-differenced autoregression of order p is introduced. Under the assumption that p approaches infinity simultaneously with the observed series length, n, the estimators are shown to be consistent both when the innovations have finite variance and when their distribution follows an infinite-variance stable law with exponent $\alpha \in (1,2)$. The relative finite sample behaviour of the estimators is investigated by a simulation study and by an application to a real data set involving ethernet traffic.

1 Introduction

Let $\{X_t\}$ be a discrete-time covariance-stationary process with 0 mean, covariance function $R(u) = E(X_t X_{t+u})$ $(t, u = 0, \pm 1, \ldots)$ and spectral density function $f(\lambda)$. Then, $\{X_t\}$ is said to exhibit long-memory with memory parameter d, $0 < d < 0.5$, if $f(\lambda)$ may be written as

$$f(\lambda) = |\lambda|^{-2d} L(1/\lambda) \tag{1.1}$$

where $L(\lambda)$ is slowly varying at infinity. A characterizing property of a long-memory time series is that its covariance function is not absolutely summable. By contrast, $\{X_t\}$ is said to be a short-memory process if its covariance function is absolutely summable, implying that $f(\lambda)$ is continuous and bounded on $[0, \pi]$.

Although the definition (1.1) above is classical, it does not readily extend to processes which are not covariance-stationary, see Heyde and Yang (1997) and Hall (1997). Nevertheless, in what follows, we use this definition for ease of exposition, though in Section 3, when considering the class of stable processes with infinite variance, we restrict the admissible values of d.