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LECTURE NOTES — MONOGRAPH SERIES

Kolmogorov-Smirnov Tests for AR Models Based on Autoregression Rank Scores

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Abstract

Tests of the Kolmogorov-Smirnov type are constructed for the parameter of an autoregressive model of order p . These tests are based on autoregression rank scores, and extend to the time-series context a method proposed by Jurečková (1991) for regression rank scores and regression models with independent observations. Their asymptotic distributions are derived, and they are shown to coincide with those of classical Kolmogorov-Smirnov statistics, under null hypotheses as well as under contiguous alternatives. Local asymptotic efficiencies are investigated. A Monte Carlo experiment is carried out to illustrate the performance of the proposed tests.

Keywords: Autoregressive models, Autoregression quantiles, Autoregression rank scores, Kolmogorov-Smirnov test, Local asymptotic efficiency.

1 Introduction

Consider the autoregressive model of order p

$$Y_t = \theta_1 Y_{t-1} + \cdots + \theta_p Y_{t-p} + \varepsilon_t, \quad t \in \mathbb{Z} \quad (1.1)$$

(AR(p) model), where $p \geq 1$ is a fixed integer, $\boldsymbol{\theta} = (\theta_1, \dots, \theta_p)'$ a p -dimensional vector of unknown autoregressive coefficients, and $\{\varepsilon_t, t \in \mathbb{Z}\}$ a process of

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