Institute of Mathematical Statistics

LECTURE NOTES — MONOGRAPH SERIES

MODELLING BY LÉVY PROCESSES

Ole E. Barndorff-Nielsen
MaPhySto[1]
University of Aarhus

1 Introduction

A considerable body of recent work uses Lévy processes to model and analyse financial time series. Section 2 provides a brief review of this work. The review is to a large extent based on two papers Barndorff-Nielsen and Shephard (2001a,b) where more detailed information may be found. See also Barndorff-Nielsen and Shephard (2001c,d,e).

The models in question aim to incorporate one or more of the main stylised features of financial series, be they stock prices, foreign exchange rates or interest rates. A summary of these stylised features, and a comparison with related empirical findings in the study of turbulence, is given in Section 3. (In fact, the intriguing similarities between finance and turbulence have given rise to a new field of study coined 'econophysics'.)

2 Lévy Processes in Finance

A Lévy process is a stochastic process (in continuous time) with independent and homogeneous increments. The study of such processes, as part of probability theory generally, is currently attracting a great deal of attention, see Bertoin (1996,1999), Sato (1999), Barndorff-Nielsen, Mikosch and Resnick (2001), and references given there.

It is by now well recognised that Brownian motion generally provides a poor description of log price processes of stocks and other financial assets. Improved descriptions are obtained by substituting Brownian motion by suitably chosen alternative Lévy processes, for instance hyperbolic Lévy motion, normal inverse Gaussian Lévy motion and, more generally, one of the

[1]Centre for Mathematical Physics and Stochastics, funded by the Danish National Research Foundation