NOTE ON A STOCHASTIC RECURSION

DAVID SIEGMUND[1]

Stanford University

The method of Yakir and Pollak (1998) is applied heuristically to a stochastic recursion studied by Goldie (1991). An alternative derivation of the Goldie's tail approximation with a new representation for the constant and some related results are derived.

AMS subject classifications: 60H25, 62L10.
Keywords and phrases: ARCH process, change-point, tail approximation.

1 Introduction

The stochastic recursion

$$R_n = Q_n + M_n R_{n-1}, \tag{1.1}$$

has been studied by a number of authors. See, for example, Kesten (1973) and Goldie (1991), who obtained an expression for the tail behavior of the stationary distribution of (1.1), and de Haan, Resnick, Rootzén and de Vries (1989), who as an application of Kesten's result obtained *inter alia* the asymptotic distribution of $\max(R_1, \cdots, R_m)$. In these studies it is assumed that $(M_1, Q_1), (M_2, Q_2), \cdots$ are independent, identically distributed and satisfy

$$P\{M_n > 0\} = 1, \;\; E(\log M_n) < 0, \;\; P\{M_n > 1\} > 0 \tag{1.2}$$

along with other technical conditions. One motive for studying (1.1) is to obtain information about the ARCH(1) process, which has been proposed as a model for financial time series. It is defined by the recursion $X_n = \{\mu + \lambda X_{n-1}^2\}^{1/2}\epsilon_n$, where $\epsilon_1, \epsilon_2, \cdots$ are independent standard normal random variables. The process X_n^2 is a special case of (1.1) with $Q_n = \mu\epsilon_n^2$, $M_n = \lambda\epsilon_n^2$. See Embrechts, Klüppelberg and Mikosch (1997) for an excellent introduction to these and related ideas, and their applications. The special case of (1.1) having $Q_n = 1$ and $E(M_n) = 1$ has also been studied in numerous papers involving change-point detection, e.g., Shiryayev (1963), Pollak (1985, 1987).

[1] This research was supported by The National Science Foundation.