INVERTING NOISY INTEGRAL EQUATIONS USING WAVELET EXPANSIONS: A CLASS OF IRREGULAR CONVOLUTIONS

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Suppose a random sample is observed from a density which is a known transformation of an unknown underlying density to be recovered. Expansion of this unknown density in a wavelet basis yields Fourier coefficients that can be reexpressed in terms of the sampled density and an extension of the adjoint of the inverse of the operator involved. This seems to yield a new approach to inverse estimation. Focusing on deconvolution optimal error rates are obtained in the case of certain irregular kernels like the boxcar that cannot easily be dealt with by classical techniques or by Donoho's (1995) wavelet-vaguelette method.

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1 Introduction

When a smooth input signal is to be recovered from indirect, noisy measurements, Hilbert space methods based on a regularized inverse of the integral operator involved usually yield optimal rates of convergence of the mean integrated squared error (MISE). Statistical theory can be conveniently developed exploiting Halmos' (1963) version of the spectral theorem (van Rooij & Ruymgaart (1996), van Rooij, Ruymgaart & van Zwet (1998)). In practice, however, the input signal often is not regular like, for instance, in dynamical systems where it might be a pulse function. In such cases the traditional recovery technique may fail to capture the local irregularities of the input. Difficulties also arise in instances where the kernel of the integral operator itself displays a certain lack of smoothness. Whenever one has to deal with irregularities, wavelet methods seem pertinent. In classical, direct estimation of discontinuous densities Hall & Patil (1995) successfully apply a wavelet expansion. For certain inverse estimation models Donoho (1995), in a seminal paper, proposes a wavelet-vaguelette decomposition for optimal recovery of spatially inhomogeneous inputs. In both papers nonlinear

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